Claims Reserving under Solvency II

Mario V. Wüthrich
RiskLab, ETH Zurich
Swiss Finance Institute Professor

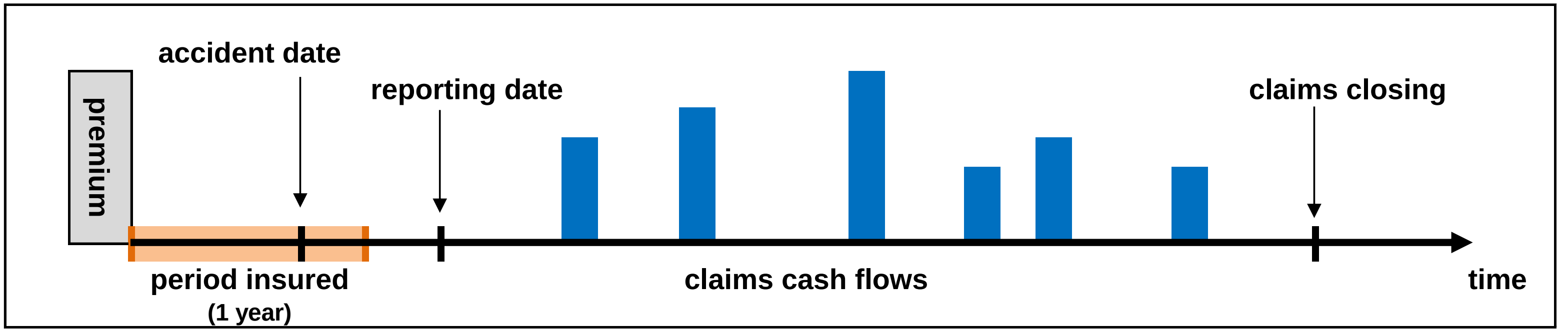
joint work with
Michael Merz (University of Hamburg)

April 21, 2016
European Congress of Actuaries, Brussels

Outline

- Claims settlement process
- Chain-ladder method
- Claims development result
- Examples

Non-life insurance claims cash flows



▷ Typically, insurance claims cannot be settled immediately at claims occurrence:

1. reporting delay (days, weeks, months or even years);
2. settlement delay (months or years).

▷ Build claims reserves to settle these future claims cash flows.

▷ Task: Predict and value all future claims cash flows.

Claims development triangle

accident year i	development year j 0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182		
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003			
2008	808	1'029	1'229	1'590	1'842	2'150				
2009	1'016	1'251	1'698	2'105	2'385					
2010	948	1'108	1'315	1'487						
2011	917	1'082	1'484							
2012	1'001	1'376					$C_{i,j}$	**to be predicted**		
2013	841									

- $C_{i,j}$ = cumulative (nominal) claims cash flows for accident year $i \in \{1, \ldots, I\}$ at development year $j \in \{0, \ldots, J\}$.

- Observations at time t:

$$\mathcal{D}_t = \{C_{i,j};\ i + j \leq t\}.$$

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Chain-ladder method

accident year i	development year j 0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182		
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003			
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2013	841									

▷ Chain-ladder (CL) algorithm is based on the assumption

$$C_{i,j+1} \approx f_j \, C_{i,j},$$

for CL factors f_j not depending on accident year i.

▷ For known CL factors f_j we can complete (predict) the lower triangle.

Stochastic models underlying the CL algorithm

- ▷ The CL algorithm is not based on a stochastic model (deterministic algorithm).
- ▷ We need a stochastic representation to quantify prediction uncertainty.
- ▷ Stochastic models introduced providing the CL reserves:
  - ⋆ Mack's distribution-free CL model (1993)
  - ⋆ Poisson and over-dispersed Poisson (ODP) model by Renshaw-Verrall (1998)
  - ⋆ Bayesian CL models by Gisler (2006), Bühlmann et al. (2009)
  - ⋆ Gamma-gamma Bayesian CL model by Merz-Wüthrich (2014, 2015)

Bayesian chain-ladder (BCL) model

Model assumptions (gamma-gamma BCL model).
Assume there are fixed variance parameters $\sigma_0^2, \ldots, \sigma_{J-1}^2$ given.

- Conditionally, given CL parameters $\boldsymbol{F} = (F_0, \ldots, F_{J-1})$:
  - ⋆ $(C_{i,j})_{j=0,\ldots,J}$ are independent Markov processes with gamma innovations
  - ⋆ with for all $1 \le i \le I$ and $0 \le j \le J-1$

$$\begin{aligned} \mathbb{E}\left[C_{i,j+1} \middle| C_{i,j}, \boldsymbol{F}\right] &= F_j \; C_{i,j}, \\ \operatorname{Var}\left(C_{i,j+1} \middle| C_{i,j}, \boldsymbol{F}\right) &= \sigma_j^2 F_j^2 \; C_{i,j}. \end{aligned}$$

- The components of $\boldsymbol{F}^{-1}$ are independent and gamma distributed.

▷ This model has the CL property: $C_{i,j+1} \approx F_j \; C_{i,j}$, for given CL factor F_j.

BCL predictor

- Predictors can be calculated explicitly in the above model for observations $\mathcal{D}_t$.

- BCL predictor at time $t \geq I > J$ for non-informative priors

$$\widehat{C}_{i,J}^{(t)} \;=\; \mathbb{E}\left[C_{i,J} \middle| \mathcal{D}_t\right] \;=\; C_{i,t-i} \prod_{j=t-i}^{J-1} \widehat{f}_j^{(t)},$$

with CL factor estimators

$$\widehat{f}_j^{(t)} = \frac{\sum_{i=1}^{(t-j-1)\wedge I} C_{i,j+1}}{\sum_{i=1}^{(t-j-1)\wedge I} C_{i,j}}.$$

CL claims prediction

accident year i	development year j 0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	*2'681*
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182	*3'424*	*3'577*
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003	*3'214*	*3'458*	*3'612*
2008	808	1'029	1'229	1'590	1'842	2'150	*2'368*	*2'534*	*2'727*	*2'848*
2009	1'016	1'251	1'698	2'105	2'385	*2'733*	*3'010*	*3'221*	*3'465*	*3'619*
2010	948	1'108	1'315	1'487	*1'731*	*1'983*	*2'184*	*2'337*	*2'514*	*2'626*
2011	917	1'082	1'484	*1'769*	*2'058*	*2'358*	*2'597*	*2'779*	*2'990*	*3'123*
2012	1'001	1'376	*1'776*	*2'116*	*2'462*	*2'821*	*3'106*	*3'324*	*3'577*	*3'736*
2013	841	*1'039*	*1'341*	*1'598*	*1'859*	*2'130*	*2'346*	*2'510*	*2'701*	*2'821*
$\widehat{f}_j^{(t)}$	1.2343	1.2904	1.1918	1.1635	1.1457	1.1013	1.0702	1.0760	1.0444	

▷ What about prediction uncertainty?

▷ Consider the conditional mean square error of prediction (MSEP)

$$\mathrm{msep}_{C_{i,J}|\mathcal{D}_t}\left(\widehat{C}_{i,J}^{(t)}\right) = \mathbb{E}\left[\left(C_{i,J} - \widehat{C}_{i,J}^{(t)}\right)^2 \middle| \mathcal{D}_t\right].$$

Conditional MSEP formula

▷ Conditional MSEP can be calculated explicitly in the above model.

Conditional MSEP for non-informative priors for single accident years i:

$$\mathrm{msep}_{C_{i,J}|\mathcal{D}_t}\left(\widehat{C}_{i,J}^{(t)}\right) = \left(\widehat{C}_{i,J}^{(t)}\right)^2 \sum_{j=t-i}^{J-1} \left[\frac{\sigma_j^2}{\widehat{C}_{i,j}^{(t)}} + \frac{\sigma_j^2}{\sum_{\ell=1}^{t-j-1} C_{\ell,j}}\right] + o\left(\frac{\sigma_\ell^2}{C_{k,\ell}}\right).$$

▷ This is the famous Mack formula (1993) up to:

- ⋆ a different variance parametrization, and
- ⋆ and a correction term of order $o\left(\sigma_\ell^2/C_{k,\ell}\right)$.

▷ Aggregation over accident years i is similar.

Example, revisited

acc.year i	$\widehat{R}_i^{(t)}$	$\sqrt{\mathrm{msep}_{C_{i,J}\mid\mathcal{D}_t}}$	in % $\widehat{R}_i^{(t)}$
2004	0		
2005	114	89	78%
2006	395	235	60%
2007	609	256	42%
2008	698	261	37%
2009	1'234	324	26%
2010	1'139	275	24%
2011	1'639	374	23%
2012	2'360	493	21%
2013	1'980	468	24%
total	10'166	1'518	15%

▷ Consider the CL reserves at time t defined by

$$\widehat{R}_i^{(t)} = \widehat{C}_{i,J}^{(t)} - C_{i,t-i},$$

and the corresponding prediction uncertainty.

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Claims development result (1/2)

- ▷ The conditional MSEP formula considers the total prediction uncertainty over the entire run-off (static view).

- ▷ Solvency considerations require a dynamic view: possible changes in predictions over the next accounting year (short term view).

- ▷ Define the claims development result of accounting year $t+1 > I$ by

$$\mathrm{CDR}_i(t+1) \;=\; \widehat{C}_{i,J}^{(t+1)} - \widehat{C}_{i,J}^{(t)}.$$

- ▷ Martingale property of $(\widehat{C}_{i,J}^{(t)})_{t \geq I}$ implies

$$\mathbb{E}\left[\mathrm{CDR}_i(t+1) \middle| \mathcal{D}_t\right] = \mathbb{E}\left[\widehat{C}_{i,J}^{(t+1)} - \widehat{C}_{i,J}^{(t)} \middle| \mathcal{D}_t\right] = 0.$$

Claims development result (2/2)

accident year i	development year j 0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	*
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182	*	
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003	*		
2008	808	1'029	1'229	1'590	1'842	2'150	*			
2009	1'016	1'251	1'698	2'105	2'385	*				
2010	948	1'108	1'315	1'487	*					
2011	917	1'082	1'484	*						
2012	1'001	1'376	*							
2013	841	*								

▷ Martingale property of $(\widehat{C}_{i,J}^{(t)})_{t \geq I}$ implies

$$\mathbb{E}\left[\mathrm{CDR}_i(t+1)\middle|\,\mathcal{D}_t\right] = \mathbb{E}\left[\widehat{C}_{i,J}^{(t+1)} - \widehat{C}_{i,J}^{(t)}\middle|\,\mathcal{D}_t\right] = 0.$$

▷ Solvency: study the one-year uncertainty

$$\mathrm{msep}_{\mathrm{CDR}_i(t+1)|\mathcal{D}_t}(0) = \mathbb{E}\left[\left(\mathrm{CDR}_i(t+1) - 0\right)^2\middle|\,\mathcal{D}_t\right].$$

One-year uncertainty formula

▷ Conditional MSEP can be calculated explicitly in the above model.

Conditional MSEP for non-informative priors for single accident years i:

$$\mathrm{msep}_{\mathrm{CDR}_i(t+1)|\mathcal{D}_t}(0) = \left(\widehat{C}_{i,J}^{(t)}\right)^2$$

$$\times \left[\frac{\sigma_{t-i}^2}{C_{i,t-i}} + \frac{\sigma_{t-i}^2}{\sum_{\ell=1}^{i-1} C_{\ell,t-i}} + \sum_{j=t-i+1}^{J-1} \alpha_j^{(t)} \frac{\sigma_j^2}{\sum_{\ell=1}^{t-j-1} C_{\ell,j}} \right] + o\left(\frac{\sigma_\ell^2}{C_{k,\ell}}\right),$$

with (credibility) weight

$$\alpha_j^{(t)} = \frac{C_{t-j,j}}{\sum_{\ell=1}^{t-j} C_{\ell,j}} \in (0,1].$$

▷ This is the Merz-Wüthrich formula (2008).

Total uncertainty vs. one-year uncertainty

Total uncertainty:

$$\mathrm{msep}_{C_{i,J}|\mathcal{D}_t}\left(\widehat{C}_{i,J}^{(t)}\right) = \left(\widehat{C}_{i,J}^{(t)}\right)^2 \sum_{j=t-i}^{J-1} \left[\frac{\sigma_j^2}{\widehat{C}_{i,j}^{(t)}} + \frac{\sigma_j^2}{\sum_{\ell=1}^{t-j-1} C_{\ell,j}} \right] + o\left(\frac{\sigma_\ell^2}{C_{k,\ell}}\right).$$

One-year uncertainty:

$$\mathrm{msep}_{\mathrm{CDR}_i(t+1)|\mathcal{D}_t}(0) = \left(\widehat{C}_{i,J}^{(t)}\right)^2 \times \left[\frac{\sigma_{t-i}^2}{\widehat{C}_{i,t-i}^{(t)}} + \frac{\sigma_{t-i}^2}{\sum_{\ell=1}^{i-1} C_{\ell,t-i}} + \sum_{j=t-i+1}^{J-1} \alpha_j^{(t)} \frac{\sigma_j^2}{\sum_{\ell=1}^{t-j-1} C_{\ell,j}} \right] + o\left(\frac{\sigma_\ell^2}{C_{k,\ell}}\right).$$

Process uncertainty, parameter estimation uncertainty and its reduction in time.

Residual uncertainty for remaining accounting years

This suggests for accounting year $t+2$:

$$\mathbb{E}\left[\mathrm{msep}_{\mathrm{CDR}_i(t+2)|\mathcal{D}_{t+1}}(0)\middle|\,\mathcal{D}_t\right]$$

$$= \left(\widehat{C}_{i,J}^{(t)}\right)^2 \left[\frac{\sigma_{t-i+1}^2}{\widehat{C}_{i,t-i+1}^{(t)}} + \left(1-\alpha_{t-i+1}^{(t)}\right)\frac{\sigma_{t-i+1}^2}{\sum_{\ell=1}^{i-2} C_{\ell,t-i+1}}\right]$$

$$+ \left(\widehat{C}_{i,J}^{(t)}\right)^2 \sum_{j=t-i+2}^{J-1} \left[\alpha_{j-1}^{(t)}\left(1-\alpha_j^{(t)}\right)\frac{\sigma_j^2}{\sum_{\ell=1}^{t-j-1} C_{\ell,j}}\right] + o\left(\frac{\sigma_\ell^2}{C_{k,\ell}}\right).$$

- ▷ This can be derived analytically and iterated!
- ▷ It allocates the total MSEP formula across different accounting periods, i.e., this provides a run-off of risk pattern.
- ▷ This was shown in Röhr (2013) and Merz-Wüthrich (2014).

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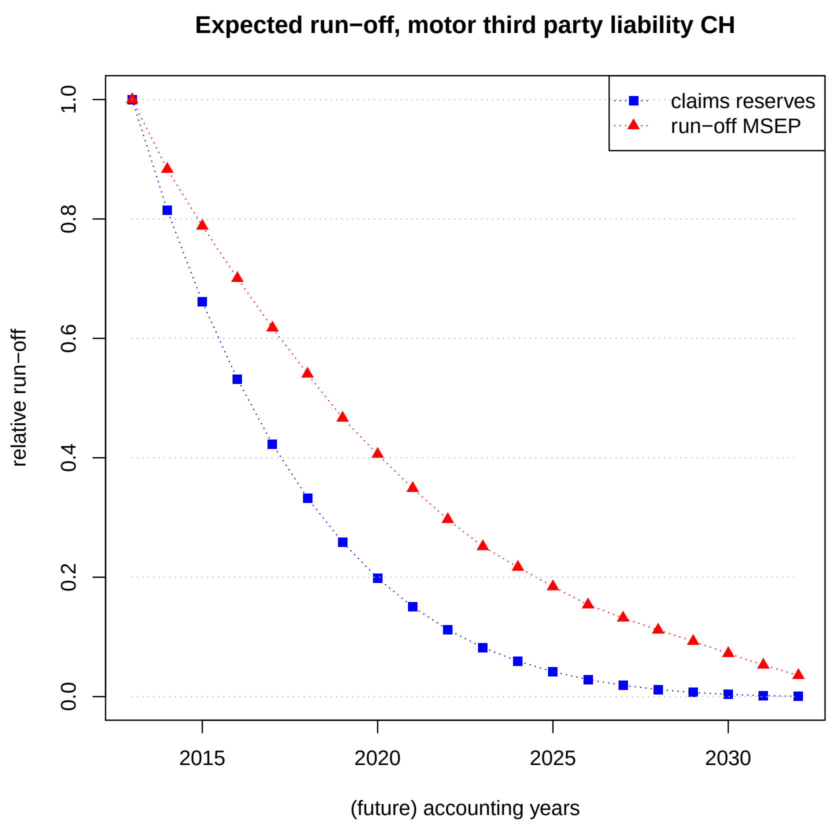
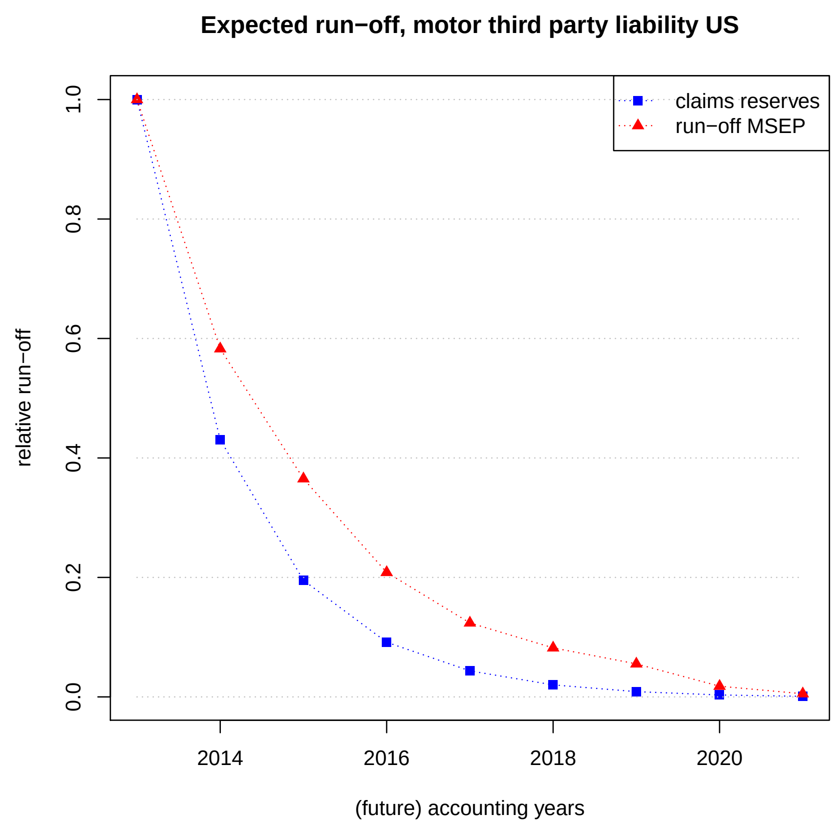
Motor third party liability: CH & US

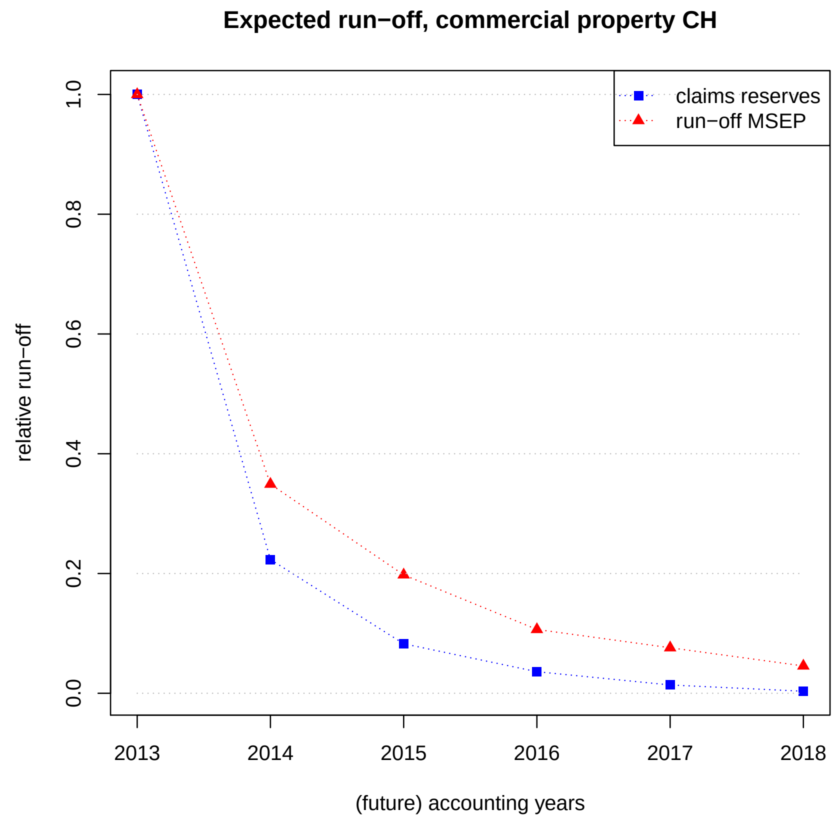


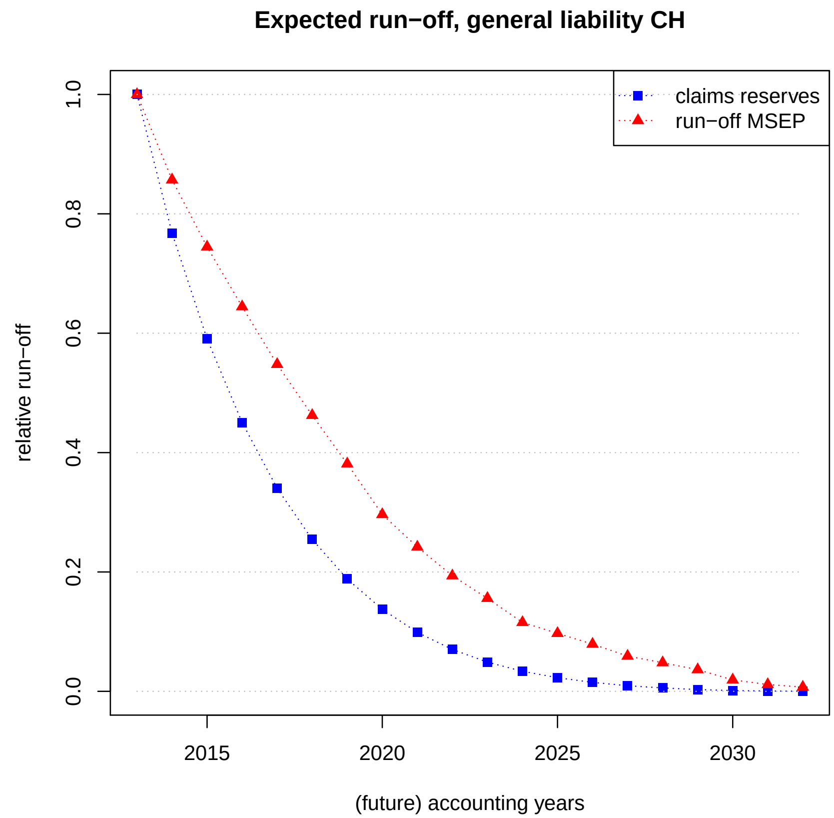


▷ Expected run-off of claims reserves is faster than the one of underlying risks.

▷ Legal environment is important for run-off.

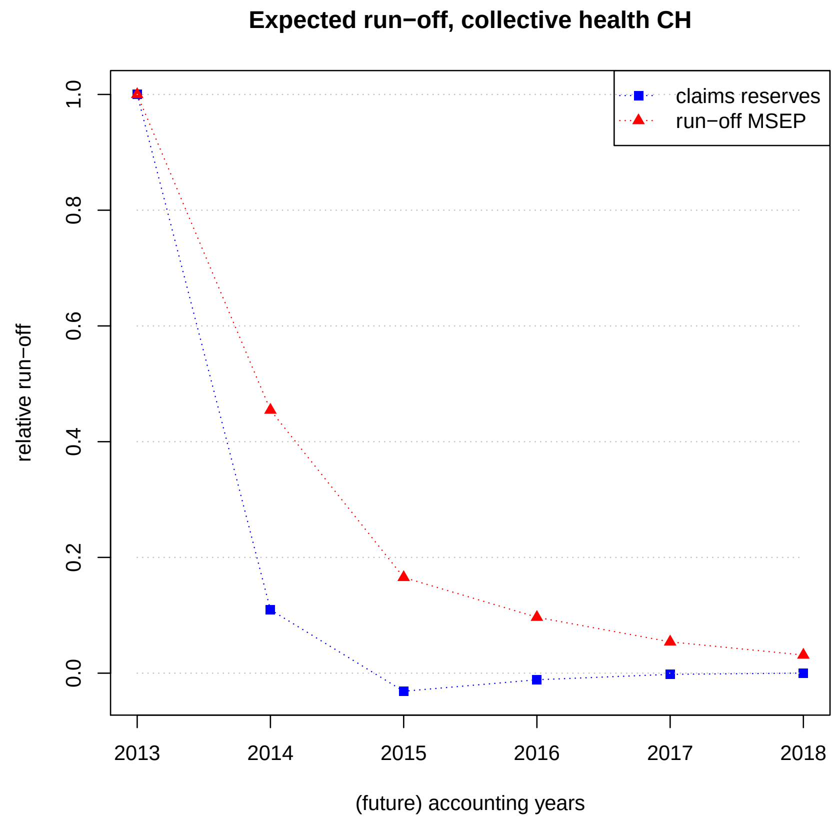
Commercial property & general liability (CH)

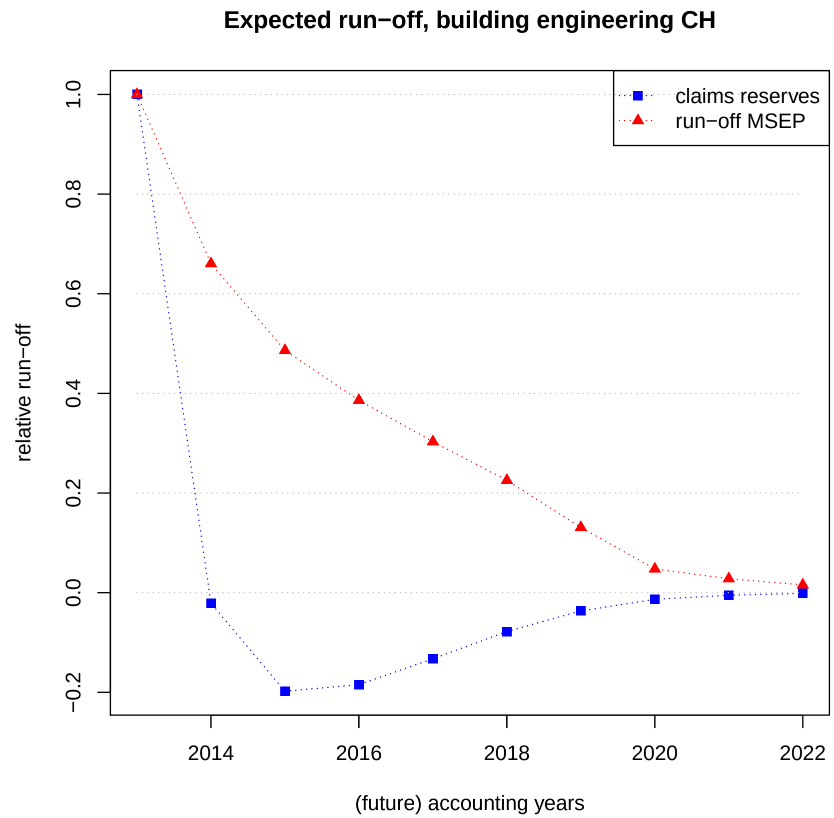




▷ Different lines of business behave differently (short- and long-tailed business).

Collective health & building engineering (CH)





▷ Subrogation and recoveries need special care.

Conclusions

- ▷ The one-year uncertainty formula was generalized to arbitrary accounting years.
- ▷ This allocates the total uncertainty formula across accounting years.
- ▷ This improves risk margin calculations under Solvency II and under the Swiss Solvency Test SST.
- ▷ Standard approximation techniques typically under-estimate run-off risk.
- ▷ Portfolio characteristics and legal environment are important for risk margins.

References

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